

[11-June-2022, Sat]

P&C-adj-1

(1) No. of blue balls = b [\rightarrow identical or distinguishable]
No. of green balls = g [\rightarrow " or "]

$(b+g)$ no. of balls are to be arranged in a row such that no two ~~balls~~ blue balls are adjacent. [$g \geq b-1$]

Find the total no. of arrangements.

Sol Let, $I_n = \begin{cases} 1, & \text{if } n \text{ balls are identical} \\ n!, & \text{if } n \text{ " " distinguishable} \end{cases}$
[my symbol] \leftarrow

First, only the green balls are arranged in a row keeping a gap between every pair of adjacent balls (so that, exactly one blue ball fits ~~in the gap~~ in the gap).

This arrangement of the green balls can be done in I_g ways.

For each of the I_g arrangements of the green balls, there are $(g+1)$ places for the blue balls to put: $(g-1)$ gaps among the green balls and 2 places at the extremities. b places out of $(g+1)$ can be selected in $\binom{g+1}{b}$ ways.

For each of the $\binom{g+1}{b}$ no. of choices, the blue balls can be arranged in their chosen places in I_b ways.

\therefore Total no. of arrangements = $I_g \binom{g+1}{b} \cdot I_b$
(Ans)

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P&C-adj-2

(2) No. of urns kept in a row = n .
No. of blue balls = b [← maybe identical or distinguishable]

Every urn can contain at most one blue ball.

The blue balls are to be put in the urns so that no two adjacent urns are occupied. [$n \geq 2b - 1$]

Find the total no. of arrangements of the blue balls.

Sol.:

Let, $g = n - b$.

Let, there be 'g' no. of identical green balls.

Then, the problem is equivalent to: finding the total no. of arrangements of the $n (= b + g)$ balls in a row, such that no two blue balls are adjacent.

By the problem - 1, it is

$$I_g \cdot \binom{g+1}{b} \cdot I_b$$
$$= \binom{n-b+1}{b} \cdot I_b \left[\because \text{the green balls are identical, } \therefore I_g = 1 \right]$$

(Ans)

[my Symbol] $I_N = 1$, if the N balls are identical
 $N!$ " " " " " " distinguishable